ASSIGNMENT: Example Assignment

1) Modelling of the Coupled Tank System
2) Design of Controllers for control of a Coupled Tank System

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RIG: Coupled Tanks
Assignment 1: Modelling of the Coupled Tank System

Modelling and characteristic analysis of a coupled tank control system

(1) Identify the dynamics and develop the mathematical model of the water level in Tank 2 \( h_2 \) with respect to the flow-in from Pump 1, \( m(t) + N(t) \).

(2) Develop a closed-loop control system for control of water level in Tank 2, including block diagram, transfer functions, and signal flow and units.

Coupled Tank Apparatus
Assignment 2: Design of Controllers for control of a Coupled Tank System

After you complete Assignment 1, you should have gained the knowledge of control system modelling and response analysis. Assignment 2 requires you to design P, PI and PID controllers (refer to lecture notes and text book).

1. Design a P controller and conduct simulation and analysis of response and stability of the control system using Matlab/Simulink;
2. Design a PI controller and conduct simulation and analysis of response and stability of the control system using Matlab/Simulink;
3. Design a PID controller and conduct simulation and analysis of response and stability of the control system using Matlab/Simulink.
Description of the UTS coupled tank system

1.1 Coupled Tank Experimental rig
The coupled tank apparatus shown in Fig.1 has two glass perspex water tanks, two pumps, two water level sensors and a reservoir. Blue food dye has been used to colour the water, to make reading the level easier.

Fig.1 Coupled Tank Apparatus
1.1.1 Pump
The two pumps are driven by PWM voltage which is supplied internally from the circuitry on the couple tank rig. The input to the pumps is a 0-5 volt DC analogue voltage, where 5 volts corresponds to the maximum flow rate. The pump had been calibrated. Fig. 3 shows the change of flow of pump #1 with voltage.
1.1.2 Level Sensors

The water level probes are of a capacitive type, which utilises the difference in the dielectric properties of air and water and hence change in capacitance to measure the water level. Changes in temperature can affect the accuracy of capacitance probes and result in offset errors. The probes are calibrated to output a DC voltage between 0 and 5 volts, with a current of 20mA (Fig.4). The range of the probes in the coupled tank rig is between 0 and 300 mm with a resolution of 1mm.
1.1.3 Coupled water tanks

Identifying the Dynamics of the Coupled Tank

- **M(i)** pump flow in
- **N(i)** disturbance flow in
- **K_1** flow/head constant (resistance out of tank 1)
- **K_2** flow/head constant (resistance out of tank 2)
- **X_1** flow rate out of tank 1
- **X_2** flow rate out of tank 2

**Flow IN tank 1** = m(t) + n(t)
**Flow OUT tank 1** = X_1(t)
**Flow IN tank 2** = X_1(t)
**Flow OUT tank 2** = X_2(t)

X_1(t) = K_1 * h_1(t)
X_2(t) = K_2 * h_2(t)

**Tank 1 equation:**

m(t) + n(t) - X_1(t) = a_1 \frac{dh_1}{dt}

**Tank 2 equation:**

X_1(t) - X_2(t) = a_2 \frac{dh_2}{dt}
Fig. 5 shows the diagram of the two tanks. Fig. 6 is an **EXAMPLE** of open loop steady state response, which shows how the water level in tank #2 changes when the flow into the system changes. **Students are required to measure the open-loop response by selecting the mode, “Open loop”**.

Two approaches exist for developing the model. The analytical approach depends on the quality of the analytical description of the plant. If an accurate model of the plant is easily developed, then a specialized controller can be designed and the range of adjustment of controller gains in this case can be usually be made small. The second approach is used when the plant is difficult to model, like this coupled tank system. Using this approach, the plant model can be estimated experimentally (Ziegler-Nichols rules).

Assume that the dynamics of the two tanks (Tanks #1 and #2) combined can be **adequately** described as a second order system (refer to lecture notes).

\[
H_2(s) = \frac{M(s) + N(s)}{k_2T_2s^2 + k_2(T_1 + T_2)s + k_2} = \frac{M(s) + N(s)}{Js^2 + as + k_2}
\]
To design a controller we need parameters $J$, $a$ and $k$ which can be calculated based on the measured response (Fig. 6) of the level in tank #2. Three methods may be used to determine $J$, $a$ and $k_2$.

$$k_2 = \Delta m/\Delta h_2.$$ 

For $J$ and $a$, we need estimates of the two time constants $T_1$ and $T_2$ that comprise the second order dynamics. The two time constants are related to the operating levels in the tanks, the difference in levels in the tanks and are directly proportional to the cross sectional area of the tanks. We define that: $t_{63\%} = T_1 + T_2 + \Sigma T_{\Delta2}$. Assuming $\Sigma T_{\Delta2}$ be of the order of 1 to 3sec.

Next, estimate $T_1$ which is the larger of the two time constants. One way of doing this is to measure the time for the level response to be 86% complete = $t_{86\%}$. Then $T_1 \approx t_{86\%} - t_{63\%}$. Then $T_2 \approx t_{63\%} - T_1 - \Sigma T_{\Delta2}$.

Another estimate \cite{1} is to draw the line of steepest slope through the response curve and locate the time where this line cuts the base line. Call this $L$. Then $T_1 \approx t_{63\%} - L$ and $T_2 \approx L - \Sigma T_{\Delta2}$.

Third method is to simply assume $T_1 = T$ and $T_2 = L$ \cite{1} \cite{2}. The models developed based on these methods are empirical models. As there are not accurate, the range of adjustment of controller gains ($K_p$, $K_i$ and $K_d$) in this case can be made large.

Finally, $a = k_2(T_1 + T_2)$ and $J = k_2 T_1 T_2$.

It should be clear that there is a fair degree of subjectivity in these estimates. The key one is $\Sigma T_{\Delta2}$ because it imposes the limit on what can be achieved. If $\Sigma T_{\Delta2}$ is underestimated then the resulting controller may be too fast for the system’s dynamics and the closed loop will be badly underdamped or even unstable.

1.2 Data Acquisition Board
The data acquisition board used is a Labjack UE9. It is connected to the LabVIEW server using an Ethernet connection.
1.3 Control Software
The LabVIEW software is used to communicate with the Labjack data acquisition board using a TCP connection.

More comments
Be sure to include the units for all quantities you measure or derive. Use seconds as the time unit for the dynamics.

Simulation software: Simulink.

References: